27. *Every 5-connected nonplanar graph contains a K_5-subdivision.* [Exercise 10.5.14]
P.D. Seymour 1974, see Seymour (2007); A.K. Kelmans 1979, see Kelmans (1993)

28. *Every 6-connected graph with no K_6-minor has a vertex whose deletion results in a planar graph.* Jørgensen (1994)

29. *No graph with more edges than vertices has a thrackle embedding.* [Exercise 10.1.11] J.H. Conway c.1968, see Woodall (1971)

30. *Every simple planar graph admits a straight-line embedding with integer edge lengths.* Kemnitz and Harborth (2001)

Extremal Problems

31. *If G is simple and $m > n(k-1)/2$, then G contains every tree with k edges.* [Exercise 4.1.9] P. Erdős and V.T. Sós 1963, see Erdős (1964)

32. *There exists a positive constant c such that $\mathrm{ex}(n, C_{2k}) \geq cn^{1+1/k}$.* [Exercise 12.2.14] Erdős (1971)

33. *If G has at most k edge-disjoint triangles, then there is a set of $2k$ edges whose deletion destroys every triangle.* Zs. Tuza 1981, see Tuza (1990)

34. *If G is a simple triangle-free graph, then there is a set of at most $n^2/25$ edges whose deletion destroys every odd cycle.* Erdős et al. (1988)

Ramsey Numbers

35. Give a constructive proof that $r(k,k) \geq c^k$ for some $c > 1$ and all $k \geq 1$. [Theorem 12.8] Erdős (1969)

36. Does the limit $\lim_{k\to\infty}(r(k,k))^{1/k}$ exist? If so, determine its value.
P. Erdős 1947, see Erdős (1961b)

37. *For every tree T on n vertices, $r(T,T) \leq 2n-2$.* Burr and Erdős (1976)

38. *For every simple graph F, there is a positive constant $\epsilon := \epsilon(F)$ such that every simple graph G which contains no induced copy of F has either a stable set or a clique of cardinality n^ϵ.* Erdős and Hajnal (1989)

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Vertex Colourings

39. (Conjecture 15.11) *Every k-chromatic graph has a K_k-minor.*
Hadwiger (1943)

40. Does a k-chromatic graph necessarily contain a K_k-subdivision when $k = 5$ and $k = 6$? [Exercise 15.4.3] Catlin (1979)

41. *Every $2k$-chromatic digraph contains a copy of every oriented tree on $k+1$ vertices.* [Theorem 4.5] Burr (1980)

42. *Every graph which can be decomposed into k complete graphs on k vertices is k-colourable.* P. Erdős, V. Faber, and L. Lovász 1972, see Erdős (1976)

43. *Every graph which can be decomposed into $k-1$ complete bipartite graphs is k-colourable.* [Theorem 2.8]
N. Alon, M. Saks, and P.D. Seymour, see Kahn (1994)

44. *The chromatic number of the weak product of two graphs is equal to the lesser of their chromatic numbers: $\chi(G \times H) = \min\{\chi(G), \chi(H)\}$.* [Exercise 14.1.18]
A. Kotzig, and Hedetniemi (1966)

45. *Let G be a k-chromatic graph which contains no k-clique, and let $k_1 + k_2$ be a partition of $k+1$ with k_1, $k_2 \geq 2$. Then there are disjoint subgraphs G_1 and G_2 of G such that G_i is k_i-chromatic, $i = 1, 2$.* [Exercise 16.3.13]
L. Lovász 1968, see Erdős (1968)

46. *The union of a 1-degenerate graph (a forest) and a 2-degenerate graph is 5-colourable.* M. Tarsi, see Klein (1994)

47. *For any graph G, $\chi \leq \lceil(\omega + \Delta + 1)/2\rceil$.* [Equation (14.2), Theorem 14.4, Exercise 14.1.14] Reed (1998)

48. *Every triangle-free graph of infinite chromatic number contains every finite tree as an induced subgraph.* Gyárfás (1975)

49. *Every nonempty graph which contains no induced odd cycle of length five or more admits a 2-vertex-colouring with no monochromatic maximum clique.*
Hoàng and McDiarmid (2002)

50. *If $\chi \geq (n-1)/2$, then $\chi_L = \chi$.* Ohba (2002)

51. *There is a constant c such that the list chromatic number of any bipartite graph G is at most $c \log \Delta$.* [Exercise 14.5.6] Alon (2000)

52. *The absolute values of the coefficients of a chromatic polynomial form a unimodal sequence.* Read (1968)